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MODELING CONSTITUTIVE BEHAVIOR OF PARTICULATE COMPOSITES UNDERGOING DAMAGE

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Abstract—A simple rate-independent phenomenological constitutive model is developed for particulate composites undergoing damage. The constitutive model is motivated by the results of a micromechanical model based on Eshelby's equivalent inclusion analysis and Mori–Tanaka's method for an elastic composite undergoing damage either by debonding or cavity formation. The micromechanical model is used to illustrate the behavior of a composite consisting of hard particles reinforcing a soft, nearly incompressible elastic matrix. The composite is assumed to behave linearly elastic in the absence of any damage. The damage accumulation is described by a single scalar internal variable, the maximum volume dilatation attained during the deformation process. Two damage functions govern the degradation of the bulk and the shear moduli in the phenomenological constitutive model. Corresponding computational algorithmic tangent moduli is derived and examples are provided to illustrate the versatility of the proposed model.

1. INTRODUCTION

Particle reinforced composites are widely used for attaining increased modulus, strength or toughness depending on the application. Such composites exhibit non-linear constitutive response due to various factors such as damage (debonding, cavity or vacuole formation, cracking), hysteresis during loading–unloading (Mullins effect), viscoelasticity (time-dependence: rate, material, damage, environment) and large strains (geometric). Such non-linear behavior is observed extensively in filled polymer or rubber products such as toughened plastics, tires, solid propellants and others. A number of studies have been performed to address one or more of the issues contributing to the non-linear behavior of these composites [see for examples Farris and Schapery (1973); Schapery (1982, 1991); Govindjee and Simo (1992)]. In this paper, we address the effect of damage by dewetting or cavity formation on the behavior of particle reinforced elastic composites.

Schematics of a two phase particulate composite and the possible damage modes when subjected to remote loading are shown in Fig. 1(a–c). We assume the particles to be spherical in shape and, under straining, damage occurs either by debonding as shown in Fig. 1(b) and/or by cavity or vacuole formation as shown in Fig. 1(c). Upon loading, at a critical strain level, the particles separate from the matrix causing dewetting. This introduces volume dilatation and results in non-linearity in the stress–strain behavior. However, for well bonded particles, cavities and cracks may form entirely within the matrix [see Cornwell and Schapery (1975)]. A typical uniaxial engineering (nominal) stress–strain curve and the corresponding volume dilatation for an elastic particulate composite (solid propellant) are shown in Fig. 2. The stress–strain response is nearly linear when there is little or no volume dilatation and the non-linearity sets in once the dilatation becomes significant. The uniaxial response and the accompanying volume dilatation of particulate composites have been investigated by a number of researchers [see for example Farris (1968); Farris and Schapery (1973); Knauss *et al.* (1973); Schapery (1982, 1987, 1991); Anderson and Farris (1988)]. It has been observed by Farris (1968) and Schapery (1987) that the stress–strain response

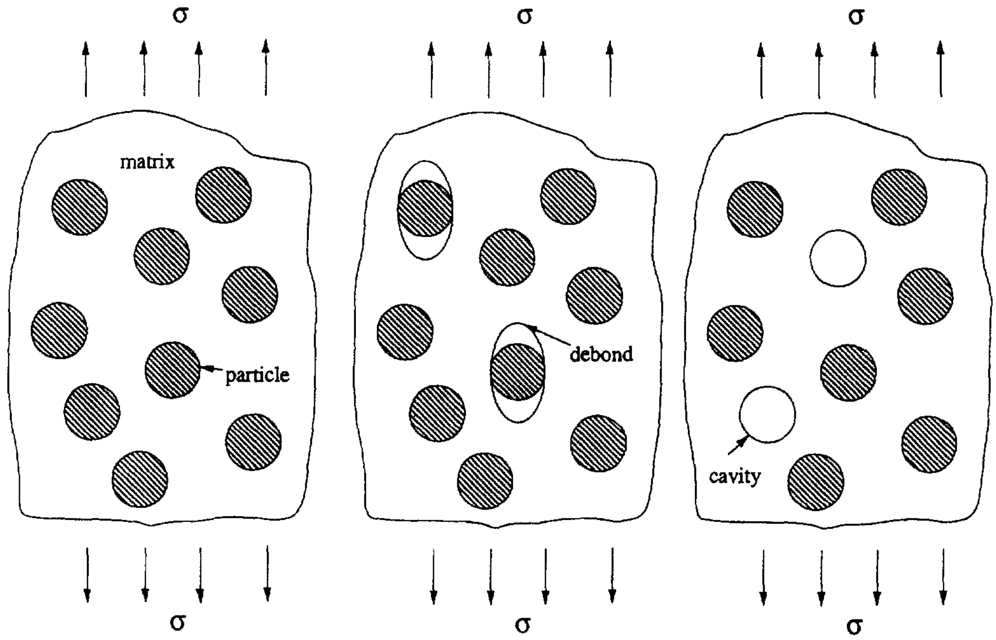

Fig. 1. Schematic of a particle reinforced composite subjected to uniaxial tension: (a) undamaged; (b) damage by dewetting (debonding) at the apex; (c) damage by cavity formation.

of particulate reinforced composites undergoing damage can be related to the constitutive response of the undamaged composite and the corresponding volume dilatation. It has also been observed that the volume dilatation is a function of the hydrostatic pressure and Schapery (1987, 1991) used a pressure–volume work term to account for the non-linearity in the stress–strain curve. Though a number of models have been available for modeling the uniaxial response of particulate composites, there are relatively few models which generalize to multi-dimensions and which could be effectively used in the engineering analysis and design of structures made of such materials. The models proposed by Schapery (1987, 1991), Simo (1987) and Anderson and Farris (1988) contain many aspects that can be used in multi-dimensional stress analysis. In our present study, we propose a simple, phenomenological, homogenized, rate-independent, constitutive model based on a single

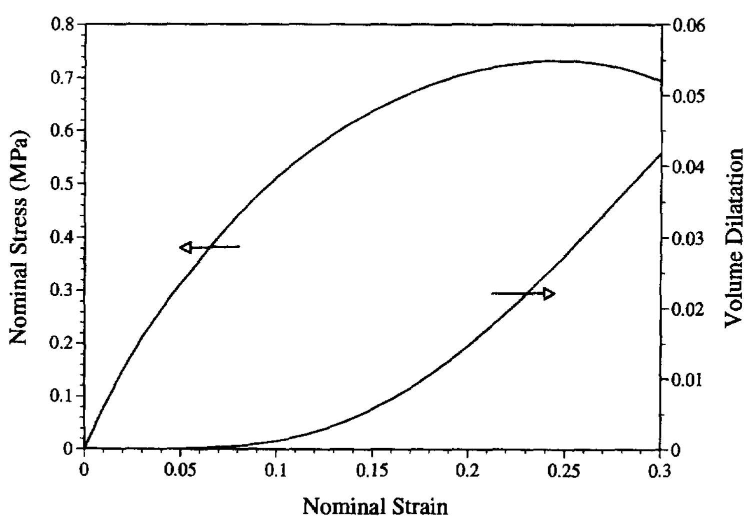

Fig. 2. Uniaxial tensile response of an elastic particulate composite undergoing damage by dewetting. Stress–strain response and dilatation as a function of strain are shown.

scalar internal variable to model particulate composites undergoing damage under generalized strain states.

In Section 2, we summarize the results from a micromechanical model based on Mori–Tanaka's method and Eshelby's equivalent inclusion analysis for an elastic composite undergoing damage either by debonding or cavity formation. Results are presented for degradation of the effective bulk and shear moduli of the composite. In Section 3, based on our observations from micromechanics and principles of continuum damage mechanics, we propose a phenomenological model to generalize the observed uniaxial response to three dimensions using a scalar parameter to govern damage evolution. The response of the undamaged composite is assumed to be governed by the classical Hooke's law. The bulk and shear moduli of the composite undergo degradation with accumulation of damage. The computational algorithmic tangent moduli for the damaged composite is derived and examples are provided to illustrate the constitutive model. In the proposed model, we have neglected the time-dependent (viscoelastic) and the finite (large) strain aspects of the problem which are of importance in studying certain classes of particulate composites such as filled elastomers. We will address the modifications to the present model which are necessary to account for the above mentioned effects in Section 4.

2. MICROMECHANICAL MODEL

When particulate composites are subjected to tensile loading, several different damage modes are possible depending on the characteristics of the interface between the matrix and the particle. We assume that the particles are spherical in shape and that under straining, damage occurs by debonding as shown in Fig. 1(b). These damage modes result in degradation of the elastic moduli of the composite materials [see for example Schapery (1986, 1991); Anderson and Farris (1988); Mochida *et al.* (1991); Tong and Ravichandran (1994)]. Of particular interest is the debonding (dewetting) between the particles and the matrix which could eventually lead to the formation of cavities (vacuole) as shown in Fig. 1(c).

Anderson and Farris (1988) and Vratsanos and Farris (1993) have presented a model where the debonding between the matrix and reinforcement has been accounted for by placing equivalent voids in the microstructure [Fig. 1(c)] and gradually increasing the volume fraction of voids. They used the differential scheme [see for example Christensen (1990)] to study the uniaxial response of such particulate composites by numerical integration. A micromechanical model developed by Schapery (1986, 1991) introduced the "two crack" model for studying the response of two phase particulate composites which undergo damage by dewetting at the apex of the spherical particles. The results from effective medium theories could be used to interpret the experimental results to obtain the bulk and shear compliances, as well as the fraction of debonded particles as a function of deformation history. In the present paper, we make use of observations from micromechanics to formulate a simple homogenized phenomenological constitutive model for particulate composites undergoing damage.

2.1. *Effective moduli*

We make use of the results from effective medium theories to predict the homogenized response of composites (consisting of an elastic matrix reinforced with elastic particles) while undergoing damage. In the present study, we make use of the results in closed form based on Eshelby's inclusion problem and Mori–Tanaka's method to account for the degradation of the moduli. By using the micromechanical results, one can study the effect of damage on the elastic moduli of the composite. We summarize the results used by Mochida *et al.* (1991) for the Young's modulus (E) and by Tong and Ravichandran (1994) for the bulk modulus (κ) of the composite undergoing damage either by debonding or void formation,

$$\frac{E}{E_{\mathrm{m}}}=\frac{1}{1+\eta_{\mathrm{p}}(1-f_{\mathrm{d}})f_0+\eta_{\mathrm{d}}f_{\mathrm{d}}f_0} \tag{1}$$

and

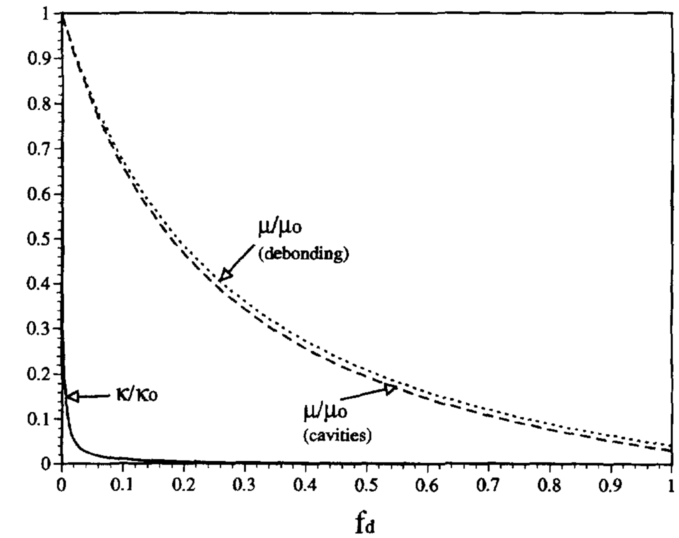

Fig. 3. Degradation of the bulk (κ) and the shear (μ) moduli as a function of the fraction of damaged particles (f_d) as predicted by the micromechanical model. The moduli are normalized with respect to the corresponding effective moduli of the undamaged composite.

$$\frac{\kappa}{\kappa_m} = \frac{1}{1+3\eta_k(1-f_d)f_0+3\eta_v f_d f_0} \tag{2}$$

where E_m and κ_m are the Young's and the bulk moduli of the matrix, respectively; f_0 is the volume fraction of the total particle reinforcements, f_d is the fraction of the damaged particles in terms of fraction of the total particles. η_p, η_d, η_k and η_v are functions of volume fractions (f_0, f_d) and elastic moduli of the matrix (E_m and κ_m) and the reinforcing particle (E_p and κ_p) [see Mochida *et al.* (1991); Tong and Ravichandran (1994)]. These factors differ depending on the mode of damage used, i.e. debonding vs cavity formation [Fig. 1(b) and (c)]. Once the Young's and the bulk moduli of the composite are known, one can compute an effective shear modulus (μ) and an effective Poisson's ratio (ν) for the composite by using standard elasticity relations,

$$\mu = \frac{3\kappa E}{(9\kappa - E)} \quad \text{and} \quad \nu = \frac{(3\kappa - E)}{6\kappa}. \tag{3a,b}$$

The effective moduli of the undamaged composite can be studied by setting $f_d = 0$ in (1) and (2). Of particular interest in dealing with polymer matrix composites is the effect of hard particles reinforcing a nearly incompressible soft matrix, i.e. E_m (matrix) $\ll E_p$ (particle). For illustrative purposes, the following material properties were used for the matrix and the particle reinforcement, $E_m = 1$ MPa, $\nu_m = 0.499$ and $E_p = 70$ GPa and $\nu_p = 0.33$. Using (3b), it is observed that the effective Poisson's ratio of the composite is dominated by that of the matrix and is close to that of the matrix for most practical values of the reinforcement. The results for effective properties of the undamaged composite obtained here coincide with the predictions by Hashin (1962). The normalized Young modulus for the composite (E/E_m) is nearly equal to that of the normalized shear modulus (μ/μ_m). These results are nearly identical for all practical values of E_m (0.1–10 MPa) and E_p (1–70 GPa) that are encountered in rubbery particulate composites.

2.2. *Degradation of moduli with damage*

To illustrate the effect of damage on the effective moduli, a composite with a large volume fraction of particles is chosen. Figure 3 shows the normalized effective bulk (κ) and effective shear (μ) moduli for $f_0 = 0.7$, i.e. 70% by volume of particles and the modulus and the Poisson's ratio values cited in Section 2.1 are used for the matrix and the reinforcement. The effective (homogenized) moduli are normalized by the respective modulus value of the composite with no damage, i.e. κ_0 and μ_0. In our present study, $\kappa_0 = 3.34\ \kappa_m$ and $\mu_0 = 6.83\ \mu_m$. The bulk modulus κ undergoes a drastic degradation (nearly two orders of magnitude) at very low levels of damage ($f_d < 0.05$) and continues to decrease with further dewetting but much more gradually. On the other hand the shear modulus (μ) degrades much more gradually with increasing damage. The shear modulus for the void case [Fig. 1(c)] is lower but very close to that of the partially debonded case [Fig. 1(b)]. The dramatic degradation of the bulk modulus upon the onset of damage can be attributed to the nearly incompressible nature of the matrix and the volume dilatation introduced by the onset of damage. It is evident that the effective shear modulus is not significantly affected by the specific assumption made regarding the nature of damage, i.e. debonding vs void formation. This is once again the consequence of near incompressibility of the matrix.

In our analysis, we have used the closed form results based on Eshelby's inclusion and Mori–Tanaka's back stress analysis to obtain the homogenized moduli for the composite undergoing damage. These results may not be accurate especially at very large volume fractions as the ones considered here and in the near incompressible range. However, one can improve these results by adopting other techniques such as the generalized equivalent inclusion method to model the problem [see for example Christensen (1990); Schapery (1991)]. It is expected that the results from other models are not expected to affect the qualitative nature of the observations made here regarding the degradation of moduli (Fig. 3). In the next section, motivated by the observations made here, a simple three-dimensional phenomenological constitutive model amenable for computational modeling is formulated to study damage evolution. The proposed model has two functions that govern the degradation of the bulk and the shear moduli of the composite material while undergoing damage. These functions can be determined readily from uniaxial experiments as a function of the proposed internal damage variable.

3. PHENOMENOLOGICAL MODEL

3.1. *Strain energy density*

From our observations in the previous section, it is apparent that in response to damage accumulation (dilatation), the bulk and the shear moduli are degraded to different extents for a given amount of damage (see Fig. 3). These results suggest that we adopt a strain energy function for the damaged material of the form

$$W(\varepsilon; D) = g(\varphi_p)\, U^0(\Theta) + h(\varphi_s)\Psi^0(\mathbf{e}) \tag{4}$$

where U^0 and Ψ^0 are the volumetric and deviatoric parts of the strain energy functions of the undamaged material and D is a generalized damage parameter. Θ and $\mathbf{e}$ are the volumetric dilatation (a scalar) and deviatoric parts of the infinitesimal strain tensor ε,

$$\Theta = \varepsilon_{kk} \quad \text{and} \quad e_{ij} = \varepsilon_{ij} - \varepsilon_{kk}\delta_{ij}/3. \tag{5a,b}$$

The phenomenological model for particulate composites undergoing damage is developed based on the concepts introduced by Kachanov (1986), Krajcinovic and Lemaitre (1987), Schapery (1987, 1991), Simo (1987) and Simo and Ju (1987). The specific functional form of g and h and as well as the choice of scalar internal variables φ_p and φ_s depend on the damage mechanism and the deformation process itself. For simplicity, we assume the damage functions $g(\varphi_p)$ and $h(\varphi_s)$ to be isotropic damage functions of scalar internal variables φ_p and φ_s, respectively. For linearly elastic materials, the functions U^0 and Ψ^0 are given by the standard expressions

$$U^0 = \kappa_0 \Theta^2/2 \quad \text{and} \quad \Psi^0 = \mu_0 e_{ij} e_{ij} \tag{6a,b}$$

where κ_0 and μ_0 are the bulk and shear moduli of the undamaged composite material. An implicit assumption made in writing the functional form for the strain energy as shown in (4) is that the composite behaves linearly elastically if it were to undergo no damage.

The stress tensor $\boldsymbol{\sigma}^0$ for the undamaged material is related to the strain tensor $\boldsymbol{\varepsilon}$ through Hooke's law for a linearly elastic material

$$\sigma^0_{ij} = \kappa_0 \Theta \delta_{ij} + 2\mu_0 e_{ij}. \tag{7}$$

The stress tensor $\boldsymbol{\sigma}$ for the damaged material for the strain energy function (4) can be written in the form

$$\sigma_{ij} = \kappa_0 g(\varphi_p) \Theta \delta_{ij} + 2\mu_0 h(\varphi_s) e_{ij}. \tag{8}$$

The dilatational and deviatoric parts of the stress tensor are altered by the factors g and h which were used to define the strain energy function W for the damaged material in terms of the strain energy of the undamaged material.

3.2. *Choice of internal variable*

The specific choice of a physical parameter for the internal variable φ is made based on our knowledge of the deformation behavior of the particulate composites discussed in Sections 1 and 2. The usual assumption made regarding the damage process in particulate composites is that the *maximum strain* (or a measure of it) attained by the material during its deformation history controls the constitutive response [see for example Farris (1968); Knauss *et al.* (1973); Schapery (1982); Gurtin and Francis (1981); Swanson and Christensen (1983)]. This assumption was used in developing constitutive models and appears to work well in the case of uniaxial loading. Schapery (1987, 1992) has outlined a methodology based on potentials like strain energy to develop a constitutive model for particulate composites undergoing damage under uniaxial straining and confining pressure. This was then combined with results from a micromechanical model to study more generalized strain states. Simo (1987) and Simo and Ju (1987) proposed to adopt the strain energy of the undamaged material as a scalar measure of the maximum strain. Subsequently, Govindjee and Simo (1992) have used the maximum stretch experienced by the material during its deformation history to be the internal variable controlling the damage process.

In the present study, we propose a single scalar internal variable for the damage parameter. The particulate composites of interest (polymeric, rubber) are nearly incompressible prior to any damage and the damage in the material due to deformation is manifested in the form of volume dilatation. In most cases, it is also observed that the response is nearly linearly elastic prior to the onset of any damage or under large superposed hydrostatic pressure [see Farris (1968)]. It appears natural to choose the volume dilatation (a scalar non-dimensional variable) or the dilatational part of the strain energy of the undamaged material U^0 to be our measure of damage in three dimensions for the particulate composites under consideration.

The volume dilatation Θ appears to be a natural choice for both φ_p and φ_s. In our subsequent discussions, the single scalar internal variable φ will be used to denote damage and we propose to use the *maximum dilatation* achieved during deformation to be the controlling damage parameter,

$$\varphi^m = \max\{\Theta\}. \tag{9}$$

3.3. *Damage criterion*

The constitutive model (8) is made complete with the damage condition

$$\Xi(\Theta, \varphi^m) = \Theta - \varphi^m \leqslant 0 \tag{10}$$

with $\Xi(\Theta, \varphi^m) = 0$ denoting a damage surface in the dilatation space. When the damage is taking place or remains the same, then $\Xi = 0$ and when loading or unloading is taking place within the damage surface, then $\Xi < 0$. The damage criterion enforces the assumption that the damage is completely controlled by the maximum dilatation φ^{m} experienced by the material during the deformation history. The evolution law for the internal variable of damage is given by a rate equation

$$\dot{\Theta} = \vartheta. \tag{11}$$

ϑ is the damage consistency parameter which is used to define loading/unloading conditions for damage using the following Kuhn–Tucker relations [see Simo and Ju (1987) for details]

$$\vartheta \geqslant 0, \quad \Xi \leqslant 0, \quad \Xi\vartheta = 0. \tag{12a,b,c}$$

When damage accumulates (increases) further during loading, i.e. $\Xi = 0$ then (12c) implies that $\vartheta > 0$, in which case the value of ϑ is determined using (11). On the other hand if $\Xi < 0$, i.e. the current dilatation Θ is less than the internal variable φ^{m}, then (12c) implies that $\vartheta = 0$. This means that no additional damage takes place during loading or unloading within the damage surface, i.e. when $\Xi < 0$.

Since the mode of damage is dewetting (debonding) between the particles and the matrix, both the bulk and the shear moduli κ and μ of the damaged material would be functions of the extent of debonding, i.e. volume dilatation. It is expected that the damage functions g and h would be different since the degradation of the bulk and shear moduli with debonding is not identical for a fixed damage level as seen in the previous section (see Fig. 3). The functions g and h can be evaluated from the uniaxial response of the composite. Using (8) for the case of uniaxial stress, i.e. $\sigma_{11} = \sigma$, $\sigma_{22} = \sigma_{33} = 0$ and denoting $\varepsilon_{11} = \varepsilon$,

$$g(\Theta) = \frac{\sigma}{3\kappa_0\Theta} \quad \text{and} \quad h(\Theta) = \frac{\sigma}{\mu_0(3\varepsilon - \Theta)}. \tag{13a,b}$$

The implicit assumptions made in arriving at the above relations are that the composite prior to damage is not incompressible, i.e. $\kappa_0 \neq \infty$ and the volume dilatation is available either from dilatometer measurements or from measurement of transverse strains ε_{22} and ε_{33}.

3.4. *Computational methodology*

From (8), together with the choice of damage parameter shown in (9), one can derive the three-dimensional algorithmic tangent moduli $\mathbf{D}$

$$D_{ijkl} = \frac{\partial \sigma_{ij}}{\partial \varepsilon_{kl}} = C_{ijkl} + \left(\kappa_0 g'(\Theta) - \tfrac{2}{3}\mu_0 h'(\Theta)\right)\varepsilon_{mm}\delta_{ij}\delta_{kl} + 2\mu_0 h'(\Theta)\varepsilon_{ij}\delta_{kl} \tag{14}$$

where

$$C_{ijkl} = \frac{\partial^2 W}{\partial \varepsilon_{ij}\partial \varepsilon_{kl}} = \left(\kappa_0 g(\Theta) - \tfrac{2}{3}\mu_0 h(\Theta)\right)\delta_{ij}\delta_{kl} + \mu_0 h(\Theta)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{15}$$

is the modulus tensor for the damaged material, i.e. $\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$. g' and h' are derivatives of the damage functions $g(\Theta)$ and $h(\Theta)$ with respect to the dilatation Θ. Note that the tangent moduli tensor D_{ijkl} is not symmetric and this asymmetry arises from the last term in (14), where the damage function h is a function of Θ or U^0. If h were to be a function of $\mathbf{e}$ or Ψ^0 rather than Θ, then the tangent moduli would be symmetric. The asymmetry in tangent moduli results in additional computational effort to obtain the solution. If the damage parameter S introduced by Schapery (1991) is used, the tangent moduli is always symmetric, however, the current damage parameter φ^{m} provides a damage criterion which approximates that when S is used.

The stiffness matrix **K** is formed at the element level using the appropriate small strain **B** matrix and the tangent modulus for the element **D** (14)

$$\mathbf{K}^e = \int_{\Omega_e} \mathbf{B}^T\mathbf{D}\mathbf{B}\,d\Omega \tag{16}$$

where Ω_e is the volume of the element. The solution is obtained by the Newton–Raphson iterative procedure.

In modeling plane problems, the case of plane strain is enforced by setting $\varepsilon_{33} = 0$, i.e. the corresponding shape function derivatives in the **B** matrix are set to be zero. In the case of plane stress, we compute the out-of-plane strain ε_{33} by setting $\sigma_{33} = 0$. In particular, this yields a relation $\varepsilon_{33} = -\bar{v}(\varepsilon_{11}+\varepsilon_{22})$ where ε_{11} and ε_{22} are the in-plane normal strain components and with $\bar{v}$ given by

$$\bar{v} = \frac{3\kappa_0 g(\Theta) - 2\mu_0 h(\Theta)}{3\kappa_0 g(\Theta) + 4\mu_0 h(\Theta)}. \tag{17}$$

However the dilatation can be written as

$$\Theta = (\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}) = (1-\bar{v})(\varepsilon_{11}+\varepsilon_{22}). \tag{18}$$

In the case of plane strain, $\bar{v}$ is set to be zero. For plane stress at the end of each iteration, the sum of the in-plane strain components is known, however the dilatation that would be used is from the previous iteration and hence the computation of out-of-plane strain component ε_{33} (and the subsequent Θ) will be inaccurate. By inspection of (17) and (18), it is clear that (17) is a transcendental equation for $\bar{v}$ and can be evaluated using the Newton–Raphson method for finding roots. This reduction proved to be extremely efficient as was seen from the quadratic convergence of the residual norm during computations. The stress components are updated with the current modulus tensor **C** (15). The iterative procedure is continued until the force and energy residual norms are below a certain set tolerance, which in our case for the force norm is set at 10^{-6}. The Kuhn–Tucker conditions (12) are enforced such that damage does not accumulate during unloading and loading within the damage surface ($\Xi < 0$). The algorithmic tangent moduli (14) is implemented in a modified version of a displacement based finite element program FEAP [see Taylor (1977)].

3.5. *Results*

The proposed homogenized constitutive model (8) for a particulate composite undergoing damage is illustrated through examples starting with the uniaxial response for a solid propellant shown in Fig. 2. Using (13a,b) with the stress–strain dilatation data shown in Fig. 2, the damage functions g and h are determined and plotted as a function of the volume dilatation in Fig. 4. The Young's modulus (E_0) and the Poisson's ratio (v_0) are 8.25 MPa and 0.499, respectively for the data shown in Fig. 2. During computations, for a given dilatation, the functions g and h are obtained by applying cubic spline interpolation to the data shown in Fig. 4.

Making use of the analytical results given in Section 2 (1)–(3), the damage function h shown in Fig. 4 was used to assess the volume fraction of the total number of particles that have undergone debonding under uniaxial tension. The computations indicate that the onset of dewetting takes place at very small strains, at around 2%, and the total number of particles that undergoes debonding increases linearly with increasing strain. For the data shown in Fig. 2, our micromechanical model predicts about 40% of the total number of particles to have undergone debonding at an axial strain of 30%.

The loading and unloading response of the material is simulated by periodically loading and unloading a specimen under uniaxial tension. The specimen is loaded, unloaded and reloaded at strains of $\varepsilon = 10\%$, 20% and 29%. The corresponding stress–strain curves and the corresponding dilatation are shown in Fig. 5. The stress–strain response and the

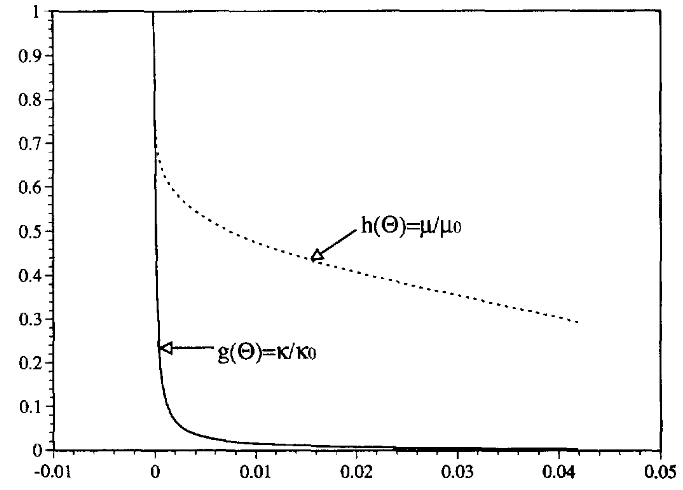

Fig. 4. Normalized bulk (κ) and shear (μ) moduli for the composite shown in Fig. 2 as a function of volume dilatation.

dilatation curve follow the experimental curve shown in Fig. 2. During unloading, the response is linear, with the material returning to the initial state with no permanent residual deformation. Upon reloading, the loading path follows the linear unloading path, rejoins the original stress–strain curve and continues to follow the stress–strain curve obtained for monotonic loading as shown in Fig. 2. Thus, the response under uniaxial stress is completely governed by the *maximum strain* attained during the deformation. This is consistent with the experimental observations and has been modeled extensively [see for example Schapery (1982)].

The response of a biaxial strip under plane stress, i.e. fixed or zero displacement in the x_1 direction and extension in the x_2 direction is shown in Fig. 6. This geometry was chosen to model the biaxial strip specimen widely used to assess the constitutive properties of particulate composites. The material exhibits a relatively softer response and increased volume dilatation with strain in comparison to the response under uniaxial tension (see Fig. 2). Such a response is consistent with experimental observations on biaxial strip specimens [see Farris and Schapery (1973)].

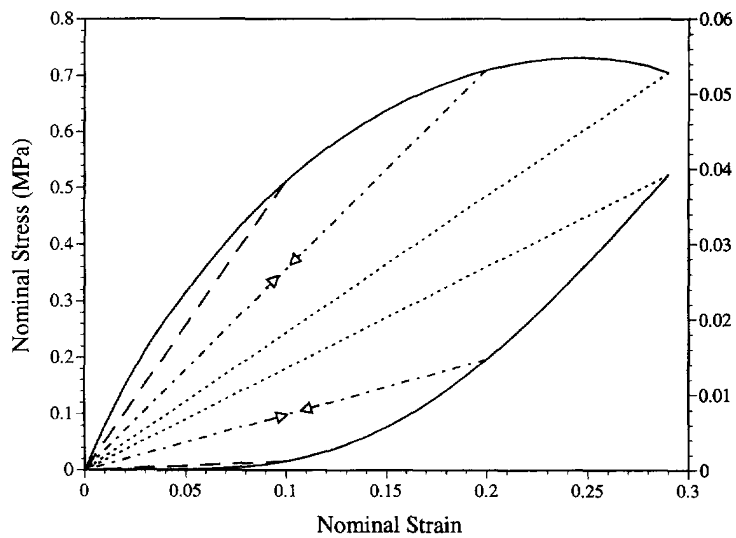

Fig. 5. Predicted loading (uniaxial tension) and unloading response for the composite shown in Fig. 2. Both the stress and the dilatation are shown as a function of the strain.

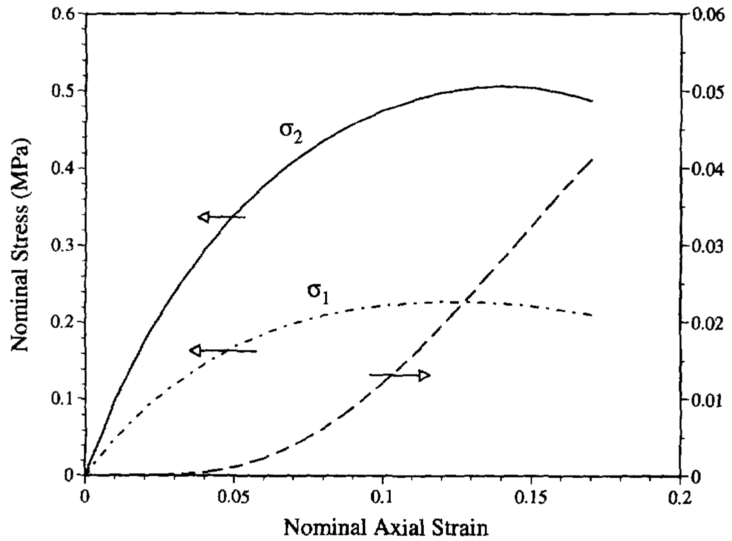

Fig. 6. Predicted response of a biaxial strip under plane stress. Normal displacement is constrained in the x_1 direction and subjected to elongation in the x_2 direction. Both the normal stress components (σ_1 and σ_2) and the volume dilatation are plotted as a function of the normal strain in the x_2 direction.

The effect of superposed hydrostatic pressure on the uniaxial response is investigated by subjecting an axisymmetric bar to radial pressure. The stress–strain response and the dilatational behavior under radial pressures of $p = 0$, 0.1, 0.25, 0.5 and 2 MPa are shown in Fig. 7. The pressures indicated are gage pressures. The linear region of the stress–strain curve increases with increasing pressure and the rate of volume dilatation decreases with increasing pressure. At relatively higher pressures, say 2 MPa, the composite exhibits nearly linear behavior for the range of strains considered here. These observations are consistent with experimental observations made by Farris (1968).

4. CONCLUSIONS AND DISCUSSION

A simple three-dimensional phenomenological constitutive model has been developed to model particulate composites undergoing damage. The model is motivated by obser-

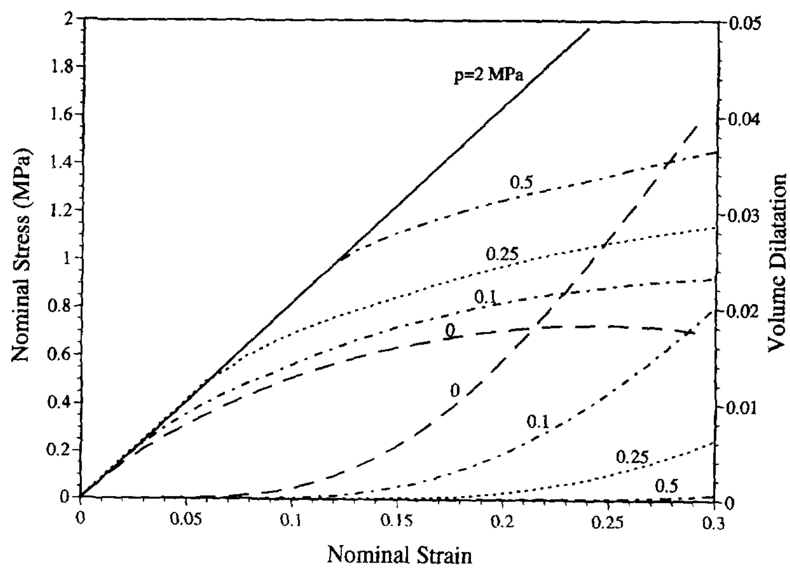

Fig. 7. Predicted response of a specimen subjected to various superposed hydrostatic pressures. Results (stress and dilatation as a function of strain) are shown for applied gage pressures, $p = 0$, 0.1, 0.25, 0.5 and 2 MPa.

vations from micromechanics regarding the degradation of bulk and shear moduli in response to damage accumulation through dewetting of reinforcing particles from the matrix. The phenomenological model is based on two damage functions which are physically related to the degradation of the moduli and the strain energy function of a linearly elastic composite. The damage parameters are functions of a single scalar internal variable, namely, the volume dilatation. Such a choice for the internal variable formulation is particularly applicable to extremely hard or rigid particles reinforcing a nearly incompressible matrix. In polymeric composites such as solid propellants, damage is usually manifested in the form of volume dilatation. The procedure to determine the two damage functions from the uniaxial measurements for a particulate composite is outlined.

The examples in Section 3 utilizing the phenomenological constitutive model indicate that the model is capable of capturing the essential experimental observations made on nearly incompressible elastic particulate composites undergoing damage. We have assumed the damage functions to be isotropic and dependent on a single internal variable. If necessary, this assumption can be relaxed to include a different description for the damage functions using the same methodology outlined in Section 3. Also, there are a number of phenomena that are relevant to such elastomeric composites that have not been included in the model. Time-dependent effects have been neglected and these can be accounted for by including viscoelasticity in the present formulation. By modifying appropriate free energy functions to include the viscous dissipation based on experimental data for relaxation, one would be able to account for the hysteresis effects observed in experiments during unloading [see for example Schapery (1982)]. Another aspect that may be important in modeling is the inclusion of geometric non-linearities or in other words, large strain effects. These effects can be accounted for by choosing an appropriate functional form for the free energy and the corresponding homogenized response function such as the generalized neo-Hookean model for the undamaged composite. In the case of finite strains, the choice for the scalar internal damage variable would be the Jacobian of the deformation gradient, which provides a measure for the volume change during deformation.

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REFERENCES

Anderson, L. L. and Farris, R. J. (1988). A predictive model for the mechanical behavior of particulate composites. *J. Polym. Engng Sci.* **28**, 522–528.

Christensen, R. M. (1990). A critical evaluation for a class of micromechanical models. *J. Mech. Phys. Solids* **38**, 379–404.

Cornwell, L. R. and Schapery, R. A. (1975). SEM study of microcracking in strained solid propellant. *Metallography* **8**, 445–452.

Farris, R. J. (1968). The character of the stress–strain function for highly filled elastomers. *Trans. Soc. Rheol.* **12**, 303–314.

Farris, R. J. and Schapery, R. A. (1973). Development of a solid rocket propellant nonlinear viscoelastic constitutive theory. Technical Report AFRPL-TR-73-50, Air Force Rocket Propulsion Laboratory, Edwards, CA.

Govindjee, S. and Simo, J. C. (1992). Transition from micromechanics to computationally efficient phenomenology: carbon black filled rubbers incorporating Mullins effect. *J. Mech. Phys. Solids* **40**, 213–233.

Gurtin, M. E. and Francis, E. C. (1981). Simple rate-independent model for damage. *J. Spacecraft* **18**, 285–286.

Hashin, Z. (1962). The elastic moduli of heterogeneous materials. *J. Appl. Mech.* **29**, 143–150.

Kachanov, L. M. (1986). *Introduction to Continuum Damage Mechanics.* Dordrecht, Boston, MA.

Knauss, W. G., Palaniswamy, K. and Smith, G. C. (1973). The application of rate theory to the failure of solid propellants. GALCIT SM Report 73-3, California Institute of Technology, Pasadena, CA.

Krajcinovic, D. and Lemaitre, J. (1987). *Continuum Damage Mechanics, Theory and Applications.* Springer-Verlag, New York.

Mochida, T., Taya, M. and Obata, M. (1991). Effect of damaged particles on the stiffness of particle/metal matrix composite. *JSME Int. J.* **34**, 187–193.

Schapery, R. A. (1982). Models for damage growth and fracture in nonlinear viscoelastic composites. *Proc. 9th U.S. Natl. Cong. Appl. Mech.* (Edited by Y. H. Pao), pp. 237–245. ASME, New York.

Schapery, R. A. (1986). A micromechanical model for non-linear viscoelastic behavior of particle-reinforced rubber with distributed damage. *Engng Fract. Mech.* **25**, 845–867.

Schapery, R. A. (1987). Deformation and fracture of inelastic composite materials using potentials. *J. Polym. Engng Sci.* **27**, 63–76.

Schapery, R. A. (1991). Analysis of damage growth in particulate composites using a work potential. *Compos. Engng* **1,** 167–182.

Simo, J. C. (1987). On a fully three-dimensional finite strain viscoelastic damage model: formulation and computational aspects. *Comp. Meth. Appl. Mech. Engng* **60,** 153–173.

Simo, J. C. and Ju, J. W. (1987). Strain- and stress-based continuum damage models—I. Formulation. *Int. J. Solids Structures* **23,** 821–840.

Swanson, S. R. and Christensen, L. W. (1983). A constitutive formulation for high-elongation propellants. *J. Spacecraft* **20,** 559–566.

Taylor, R. L. (1977). Computer procedures for finite element analysis. In *The Finite Element Method* (Edited by O. C. Zienkiewicz). McGraw-Hill, London.

Tong, W. and Ravichandran, G. (1994). Effective elastic moduli and characterization of a particulate-reinforced metal matrix composite with damaged particles. *Comp. Sci. Tech.* (in press).

Vratsanos, L. A. and Farris, R. J. (1993) A predictive model for the mechanical behavior of particulate composites, part I: model derivation. *Polym. Engng Sci.* **33,** 1458–1465.